

## Lenses and Mirrors

*And now for the sequence of events, in no particular order.*

*— Dan Rather*

### Overview

We will now study formation of images by lenses and mirrors, and the use of these in simple optical instruments. For this purpose the ray approximation is sufficient.

A lens is a device that uses refraction to bend light rays; a mirror uses reflection. The purpose in bending the rays, in most cases, is to form an image of an object, that is, an optical replica of the object.

The lenses and mirrors we discuss will be assumed to have either plane or spherical surfaces, which simplifies the geometry. For the most part, we will further assume that the rays that are used make small angles with the symmetry axis of the device. This is the **paraxial ray approximation** and it allows derivation of simple formulas for locating and describing images. Finally, we will assume that the indices of refraction of lenses are independent of the wavelength of light, ignoring the effects of dispersion.

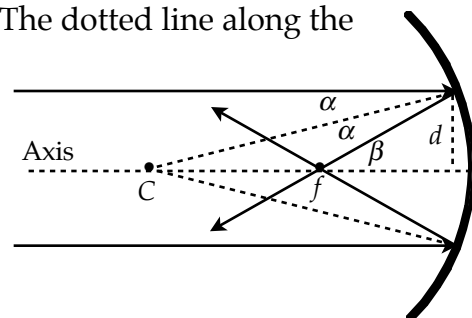
Some "aberrations" that arise from violations of these approximations will be discussed.

### Mirrors

We consider mirrors made of a conducting material (so the reflectivity is equal to 1) in the shape of part of a sphere. If the mirror surface is concave, the mirror is called "converging" or "positive" (for reasons to be made clear); if the surface is convex, the mirror is called "diverging" or "negative".

Consider first a concave mirror, shown in a side view. The dotted line along the sphere's diameter is the symmetry axis. Consider two incident rays, parallel to the axis and close to it, so the angles involved in the reflection are small.

Point C is the center of the spherical surface, which has radius  $R$ . After reflection the two rays cross each other at a point on the axis. This is the **focal point** of the mirror. Its distance  $f$  from the mirror (the **focal length**) is obtained by simple geometric arguments.



The two right triangles with opposite side  $d$  give (using the paraxial ray assumption that both angles are small)

$$\alpha \approx \tan \alpha = d / R, \quad \beta \approx \tan \beta = d / f$$

Since the angle of reflection is equal to the angle of incidence, we see that  $\beta = 2\alpha$ . Thus we find a simple formula for  $f$ :

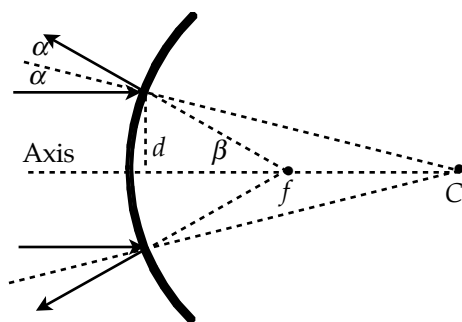
Focal length of a spherical mirror	$f = R / 2$
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In this case the parallel rays “converge” to the focal point, which is why the mirror is called “converging.” Its radius  $R$  and its focal length  $f$  are assigned *positive* values in this case, which is why it is also called a “positive” mirror.

Next consider a convex mirror, as shown. The center of the sphere is on the side opposite to that where the light impinges and is reflected.

The reflected rays (coming from the left of the mirror) diverge *as though* they had come from a point behind the mirror. This is the focal point, but it is **virtual**.

The reflected light does not actually come from this point, but an observer whose senses and brain process the information from the reflected rays will interpret them as if they originated from that point, because the rays would be the same if they had.



Our sense of where an object is located comes from the capacity of our brain to project back diverging rays to their source, whether that source is “real” or “virtual”. It is only because of other information we possess that we can decide whether the source is real or virtual.

The same geometric argument used in the converging case leads to the same formula for  $f$  in terms of  $R$ , but here by convention we assign to both quantities negative numbers. This mirror is called “negative.” Because it diverges parallel incident rays, it is also called “diverging.”

In the sign convention we use, positive distances represent “real” things, while negative distances represent “virtual” things. For mirrors, centers of curvature and focal points in front of the mirror are “real” and  $R$  and  $f$  are positive; those points behind the mirror are “virtual” and  $R$  and  $f$  are negative.

## Image formation by mirrors

Images are of two types:

**Real images.** Rays from a point in the object are converged by the optical system at a point in space, which is the corresponding point in the real image.

**Virtual images.** Rays from a point in the object are diverged by the optical system as though they had emanated from a point in space, which is the corresponding point in the virtual image.

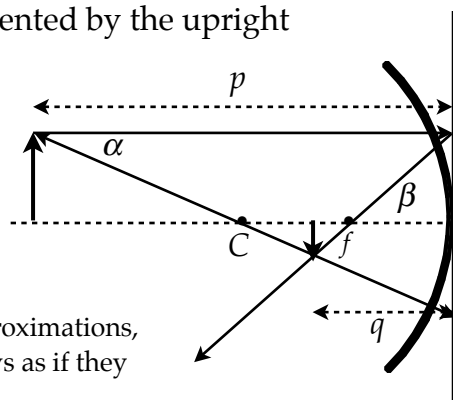
To locate the image point formed by a mirror, one uses **principal rays**:

1. A ray from the object point, passing through or toward the center of curvature. This ray strikes the mirror at normal incidence and is reflected straight back.
2. A ray parallel to the symmetry axis. For a positive mirror, this ray is reflected through the real focal point. For a negative mirror it is reflected away from the virtual focal point.
3. A ray passing through or toward the focal point, and emerging parallel to the axis, can also be used.

Any two of these are sufficient to locate images.

We consider a converging mirror, with an object (represented by the upright arrow) located at a distance  $p$  from the mirror, greater than the focal length. The two principal rays from the tip of the object converge to form the tip of the image (represented by the small inverted arrow) located at distance  $q$  from the mirror. Since the rays do actually converge there, this is a real image.

To emphasize that we are using paraxial rays and small angle approximations, we represent the mirror by a vertical straight line and draw the rays as if they impinge on that line.



Let the height of the object be  $h$  and that of the image  $h'$ . Then from the right triangles in the drawing and the small angle approximations, we find after a short calculation:

Image location formula	$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$
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We also find that  $h'/h = q/p$ . This ratio gives the **lateral magnification** of the image relative to the object. It is customary to define this magnification with a negative sign to denote the fact that a real image is inverted relative to the object. Thus we have

Lateral magnification	$m = -\frac{q}{p}$
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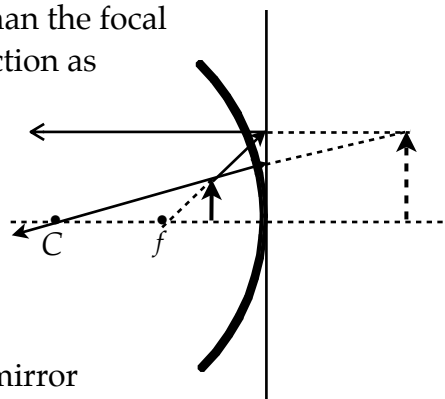
In the case shown,  $q < p$  so the image is smaller than the object.

This is because  $p > 2f$  in the case we have drawn. One can show from the image location formula that if  $f$  is positive and  $p > 2f$  then  $q$  is positive and  $q < p$ .

Now suppose the small inverted arrow were the object. Then the rays would be the same, but reversed in direction. The large upright arrow would then become the image. This is an example of the "principle of reversibility", which says that reversing the directions of all the rays gives another possible optical situation.

In that case the object distance is between  $f$  and  $2f$ , and the image distance is greater than  $2f$ . The image is real, inverted (relative to the object) and enlarged.

Things are different if the object is closer to the mirror than the focal point. Shown is such a case. The rays diverge after reflection as though they had come from the tip of the dashed arrow *behind* the mirror. This is a *virtual* image. The image distance  $q$  is now *negative*. The image is erect (relative to the object) and enlarged. The details can be calculated using the two formulas given above, which are still valid.

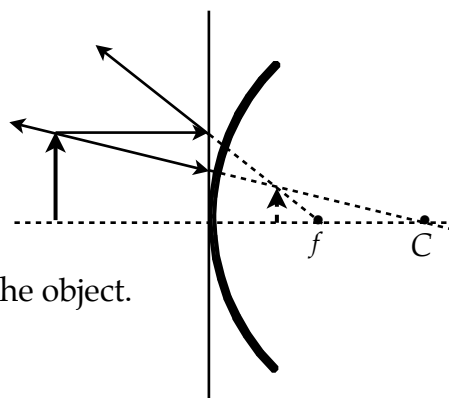


An object placed exactly at the focal point of a positive mirror results in reflected rays that are all parallel to the axis. The reflector mirror in a searchlight is an example of this. Conversely, an object at essentially infinite distance will produce an image at the focal point. The reflector mirrors in astronomical telescopes are examples.

A negative mirror always gives a virtual image of a real object. Shown is a case. The virtual image is erect and reduced. The details can be calculated from the formulas, but one must remember that  $f$  is negative.

Surveillance mirrors in shops are of this type, as are the outside right mirrors in modern automobiles.

Note that the image is always closer to the mirror than the object.



## Thin lenses

A lens is a device made of a transparent material with index of refraction different from that of the medium from which the light impinges. Focal points and images are produced by refraction at the surfaces. Our analysis will be restricted to lenses with spherical surfaces, and with thickness that are small compared to the radii of curvature of the surfaces. These are "thin" lenses.

One defines focal points for lenses in a way similar to that for mirrors. If, after passing through the lens, incident rays parallel to the axis are converged at a point, then that point is the focal point, and we have a converging or positive lens; its focal length (the distance from the lens to the focal point) is positive. If, after passing through the lens, the parallel rays diverge as though coming from a point on the same side of the lens as

the incident light, then we have a diverging or negative lens. The focal point is where the diverging rays appear to have come from, and the focal length is negative.

For paraxial rays one can show, using the law of refraction and small angle approximations, that the focal length is given by the following formula:

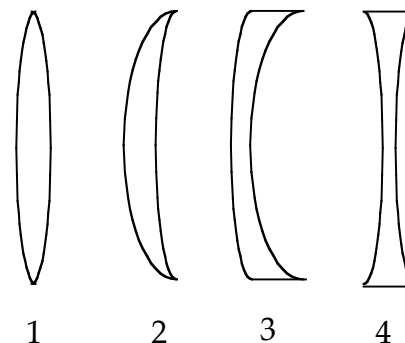
Lens maker's formula	$\frac{1}{f} = \left( \frac{n}{n_0} - 1 \right) \cdot \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$
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Here  $n$  is the index of refraction of the substance from which the lens is made (usually glass or plastic),  $n_0$  is the index of refraction of the transparent medium on either side of the lens (if it is air then  $n_0 \approx 1$ ).  $R_1$  and  $R_2$  are the radii of the two lens surfaces, for which there are sign conventions.

For the formula as given, the sign conventions are as follows: As one follows the light through the lens, one encounters the first surface and then the second; if a surface is convex (bulging toward the incoming light) its radius is positive, otherwise negative. Various texts may use different conventions, and therefore have different signs in the formula.

For ordinary glass lenses in air, the following are typical shapes in cross section. The signs are as follows:

1.  $R_1 > 0, R_2 < 0, f > 0$ .
2.  $R_2 > R_1 > 0, f > 0$ .
3.  $R_1 > R_2 > 0, f < 0$ .
4.  $R_1 < 0, R_2 > 0, f < 0$ .



One sees a simple rule here: lenses that are thicker in the middle have positive focal length; those that are thinner in the middle have negative focal length.

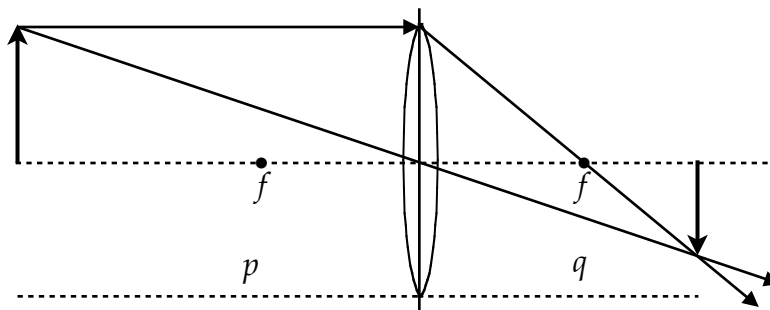
Shapes 2 and 3 are typical of corrective eyeglass lenses, 2 to correct farsightedness or enhance accommodation, 3 to correct nearsightedness.

## Image formation with lenses

The procedure for locating images with lenses is similar to that for mirrors. The commonly used principal rays are:

1. A ray from the object point to the center of the lens, where the two surfaces are parallel. This ray passes essentially straight through.
2. A ray from the object point parallel to the axis. This is refracted through the focal point for a positive lens, or away from it for a negative lens.
3. A ray passing through or toward a focal point emerges parallel to the axis.

Shown is a real image formed by a positive lens, with object beyond the focal point.

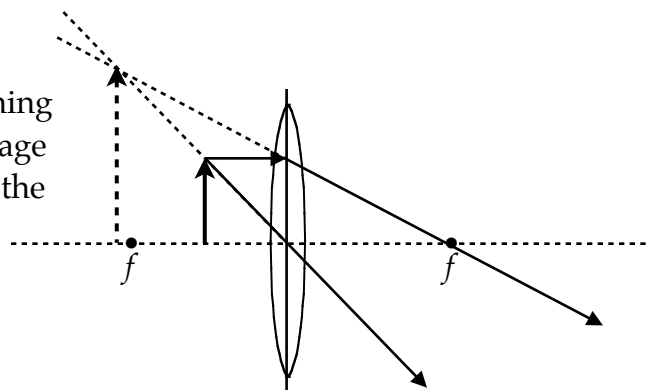


The analysis (in the paraxial ray approximation) gives the same formulas for location of the image (and for lateral magnification) as we had for mirrors.

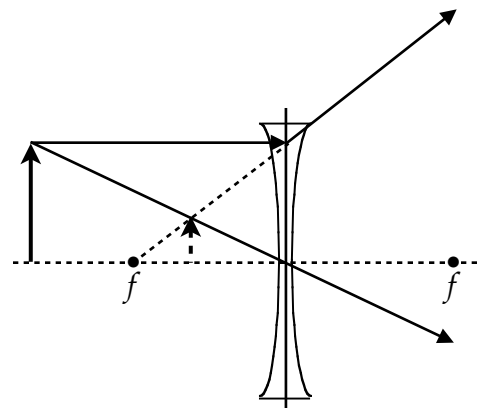
In the case shown above, the object distance is greater than  $2f$  so the image distance is less than  $2f$ , and the image is real, inverted and reduced. (To form a real and enlarged image, the object distance must be between  $f$  and  $2f$ .)

Most cameras form real, reduced images of objects more distant than  $2f$ . Slide projectors form real, inverted and enlarged images on a distant screen, of objects between  $f$  and  $2f$ .

As with the positive mirror, an object placed closer to the lens than  $f$  will form a virtual image. The rays are as shown, forming an upright, enlarged virtual image. The image distance is negative. An example of this is the enlarged virtual image produced by a magnifying glass.



Like a negative mirror, a negative lens produces only virtual images of real objects. The images are erect and reduced, located closer to the lens than either the object or the focal point, as the diagram shows. In this case, both  $f$  and  $q$  are negative.



## Aberrations

The formulas we have discussed are simple because of the approximations we made. Deviations from them are to be expected, and do occur. They are called **aberrations**.

Some of the aberrations have to do with inadequacy of the geometric approximations used in deriving our simple formulas. Three of the most common problems are these:

- If the actual system has incident rays that are not paraxial, i.e., whose distance from the axis is not small compared to the radii of surfaces of lenses and mirrors, then our claim that all rays parallel to the axis will be brought to a single focal point is not valid. The resulting blurring of images is called *spherical aberration*. It can be reduced by putting a small aperture next to the lens, permitting only paraxial rays to enter. Of course this limits the amount of light admitted, and hence the brightness of the image. It also increases the importance of diffraction effects, as we will see later.
- If the optical system is not really axially symmetric, we have an aberration called *astigmatism*. This is a common defect of the eye.
- In the case of lenses, we have ignored the slight variation of the indices of refraction with wavelength (dispersion). As a consequence of dispersion, light waves of different wavelengths have different focal points. The resulting blurring of images is called *chromatic aberration*.

Professional optical systems often use multi-element systems to correct for these aberrations.